1. (i) Given that the roots of the equation $x^2 - x + 2 = 0$ are α and β, find the quadratic equation whose roots are $\frac{1}{1+\alpha^2}$ and $\frac{1}{1+\beta^2}$.

(ii) Given that the polynomial
$$P(x) = (2x-1)(x-3)Q(x) + 12x - 8,$$
of degree 3, is exactly divisible by $x - 1$ and that $P(0) = 10$, find $Q(x)$.

(10 marks)

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2. Given the matrices **M** and **N**, where
$$\mathbf{M} = \begin{pmatrix} 2 & 1 & 0 \\ 1 & -1 & 1 \\ 3 & 1 & 0 \end{pmatrix} \text{ and } \mathbf{N} = \begin{pmatrix} -1 & 0 & 1 \\ 5 & 0 & -2 \\ 6 & 3 & -3 \end{pmatrix},$$
find matrix product **MN** and **NM**,
Hence find $\mathbf{M}^{-1}$, inverse of **M**

The transformation represented by the matrix **M** maps the points A, B and C to the points $(1, 0, 6)$, $(0, 5, 3)$ and $(1, 0, 1)$ respectively.
Find the coordinates of A, B and C.

(8 marks)

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3. (i) Given that the function $f(x) = x^3$ is differentiable in the interval $(-2, 2)$, use the mean value theorem to find the value of x for which the tangent to the curve is parallel to the chord through the points $(-2, 8)$ and $(2, 8)$.

(ii) Express in the form $y = f(x)$, the general solution of the differential equation $x\frac{dy}{dx} = y(1 - y^2)$.

(12 marks)

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4. (i) Use De moivre's theorem to express $\cos 4\theta$ in terms of $\cos\theta$.

(ii) Given that $z_1 = 2 + i$, $z_2 = -2 + 4i$ and $\frac{1}{z_3} = \frac{1}{z_1} + \frac{1}{z_2}$, find z_3.

(12 marks)

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5. The position vectors of the points A, B and C are **a**, **b** and **c** respectively, where
$$\mathbf{a} = 3\mathbf{i} + 6\mathbf{k},\ \mathbf{b} = 5\mathbf{j} + 3\mathbf{k} \text{ and } \mathbf{c} = \mathbf{i} + \mathbf{k}.$$
Find
(i) the vector product $\overrightarrow{AC} \times \overrightarrow{AB}$,
(ii) the vector equation of the plane ABC.

(8 marks)

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6. Express $f(\theta) = 8\cos\theta - 15\sin\theta$ in the form $r\cos(\theta + \alpha)$, where r is positive and α an acute angle.
Hence, find
(i) the general solution of the equation $80\cos\theta - 150\sin\theta = 13$,
(ii) the maximum and minimum value of $\frac{5}{f(\theta)+3}$.

(9 marks)

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7. (i) Express $f(x) = \dfrac{5x-3}{(x+1)(x+3)}$, in partial fractions.
   Hence, evaluate $\int_3^5 f(x)dx$.

(ii) Given that $f(x) = 5x^2 - 4\sqrt{x} - 6$, $x > 0$ and taking 1.5 as a first approximation to the root of $f(x) = 0$, use the Newton-Raphson procedure to obtain, to three decimal places, a second approximation to the root of the equation.

(10 marks)

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8. (i) Find the set of values of x for which $\dfrac{x+2}{x-1} < 3$.

(ii) Given the function f, where $f(x) = \dfrac{2x-4}{x+2}$, $x \neq -2$,
(a) find the range of f.
(b) sketch the graph of f.

(10 marks)

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9. (In this question, you are required to work throughout with two decimal places.)

The table below shows the values of x and y obtained in a certain laboratory work.

x	2.659	4.801	6.248	9.708	17.595	20.96
y	10.317	6.569	5.63	4.512	3585	3.38

The variables x and y are connected by the equation $y - 2 = b(x-1)^a$.
By drawing a suitable graph of $\log_{10}(y-2)$ against $\log_{10}(x-1)$, estimate the values of the constants a and b.

(10 marks)

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10. (i) Find the term independent of x in the expansion of $(x^2 + \frac{1}{2x})^6$.

(ii) A geometric progression with positive terms has the sum of its first two terms as $\frac{20}{3}$ and its sum to infinity is 12.
Find
(a) the first term and the common ratio of the progression.
(b) the sum of the first three terms.

(12 marks)